MECHANICAL PROPERTIES OF SOLIDS

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ELASTICITY:- If a body regains its original dimension after the removal of deforming force, it is said to be elastic body and this property is called elasticity.

If a body regains its original dimension completely and immediately after the removal of deforming force, it is said to be a perfectly elastic body.

The nearest approach to a perfectly elastic body is quartz fibre.

Plasticity:- If a body doesn't regain its original dimension even after the removal of deforming force, it is said to be a plastic body and this property is called plasticity.

If a body does not show any tendency to regain its original size and shape i.e. dimension even after removal of deforming force, it is said to be perfectly plastic body.

Putty and Paraffin wax are nearly perfectly plastic body.

Stress:-

The internal restoring force set up per unit area of cross-section of the deformed body is called stress.

$$\text{i.e., Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

The restoring force is equal and opposite to the external deforming force, therefore

$$\text{Stress} = \frac{\text{Deforming}}{\text{Area}}$$

$$\therefore \boxed{\text{Stress} = F/A}$$

The S.I. unit of stress is N/m^2. The dimensional formula of stress is $[M^1L^{-1}T^{-2}]$

Types of stress:-

(i) **Tensile stress:-** It is the restoring force setup per unit cross-sectional area of a body when the length of the body increases in the direction of deforming force.

(ii) Compressional Stress:- It is the restoring force setup per unit cross-sectional area of a body when its length decreases under a deforming force.

(iii) Tangential or Shearing Stress:- When a deforming force acts tangentially to the surface of a body, it produces a change in shape of the body. The tangential force applied per unit area is equal to the tangential stress.

Strain:- The ratio of change in any dimension produced in the body to the original dimension is called strain.

i.e, $\text{Strain} = \dfrac{\text{Change in dimension}}{\text{original dimension}}$

Strain is the ratio of two similar quantity so it has no units and no dimensions.

Types of Strain:- Following are the three types of strain -

(i) Longitudinal Strain:- It is defined as the increase in length per unit original length, when the body is deformed by external forces.

i.e, $\text{Longitudinal Strain} = \dfrac{\text{Change in length}}{\text{original length}}$

$= \dfrac{\Delta l}{L}$

(ii) Volumetric Strain:- It is defined as the change in volume per unit volume (original) when the body is deformed by external forces.

i.e, $\text{Volumetric Strain} = \dfrac{\text{Change in volume}}{\text{original volume}}$

$= \dfrac{\Delta V}{V}$

(iii) Shear Strain:- It is defined as the angle through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.

i.e, Shear Strain $= \theta$

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$$= \frac{\text{Relative displacement between two } \parallel \text{ planes}}{\text{Distance between parallel planes}}$$

Elastic limit :-

The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit.

Hooke's law :-

It states that within elastic limit, the stress is directly proportional to strain.

Thus within elastic limit.

$$\text{Stress} \propto \text{Strain}$$

$$\text{or, Stress} = E \times \text{Strain}$$

Where E is a constant, called modulus of elasticity or coefficient of elasticity of the material. It's value depends on the nature of the material of the body and the manner in which it is deformed.

$$\therefore E = \frac{\text{Stress}}{\text{Strain}}$$

Modulus of elasticity

Modulus of elasticity is the reference to measure the elastic property of material of body.

It may be defined as the ratio of stress to the co-responding strain, within the elastic limit.

$$\text{i.e, } E = \frac{\text{Stress}}{\text{Strain}}$$

The S.I. unit of modulus of elasticity is N/m^2 and it's dimensions are $[M^1 L^{-1} T^{-2}]$

Types of modulus of elasticity —

Corresponding to the three types of strain, we have three important moduli of elasticity:

(i) **Young's modulus :-**

It is the ratio of normal stress to the longitudinal strain within elastic limit.

$$\text{i.e, } Y = \frac{\text{Normal Stress}}{\text{Long. Strain}}$$

$$= \frac{F/A}{}$$

$$or,\ Y = \frac{FL}{A\Delta l}$$

If wire has circular section of radius r, then

$$\boxed{Y = \frac{FL}{\pi r^2 \times \Delta l}}$$

(ii) Bulk modulus of elasticity :— It is the ratio of normal stress to the volumetric strain within elastic limit.

$$i.e.\ K = \frac{\text{Normal Stress}}{\text{Volumetric Strain}}$$

$$or,\ K = -\frac{F/A}{\Delta V/V}$$

$$or,\ K = -\frac{FV}{A \times \Delta V}$$

$$\therefore \boxed{K = -\frac{PV}{\Delta V}} \quad \left[\because \frac{F}{A} = P\right]$$

Note:- The reciprocal of bulk modulus of a material is called compressibility.

$$i.e.\ \text{Compressibility} = \frac{1}{K}$$

(iii) Modulus of rigidity :— It is the ratio of tangential stress to the shear strain within elastic limit.

$$i.e.\ \eta = \frac{\text{Tangential Stress}}{\text{Shear Strain}}$$

$$or,\ \eta = \frac{F/A}{\theta}$$

$$\therefore \boxed{\eta = \frac{F}{A\theta}}$$

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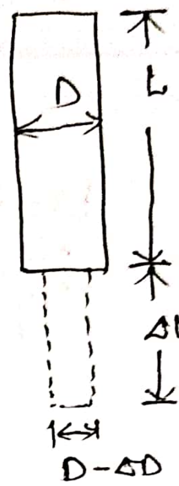
Poisson's Ratio:— Within elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

i.e. $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

Consider the length of the loaded wire increases from L to $L+\Delta L$ and its diameter decreases from $D-\Delta D$. Then

Longitudinal strain $= \frac{\Delta L}{L}$



and lateral strain $= -\frac{\Delta D}{D}$

So, Poisson's ratio is

$$\sigma = \frac{-\Delta D/D}{\Delta L/L}$$

$$\therefore \boxed{\sigma = -\frac{L\times\Delta D}{D\times\Delta L}}$$

The negative sign indicates that longitudinal and lateral strains are in opposite sense.

As the Poisson's ratio is the ratio of two strains, it has no units and no dimensions.

Note:: If α is the longitudinal strain per unit stress and β is the lateral strain per unit stress, then Poisson's ratio is

$$\boxed{\sigma = \frac{\beta}{\alpha}}$$

Elastic potential energy stored in a stretched wire:—

When deforming force is applied on a wire then restoring force set up in a wire and work has to be done by applied force in deforming a body against restoring force, this work done stored up in the wire in the form of potential energy.

Consider a wire of length L and cross-sectional area A. Let F be the tensile deforming force applied to a wire then wire be elongated and length of wire increased by l. So

$$\therefore F = \frac{YAl}{L} \quad \text{--- (i)}$$

If x is the elongation at any instant during the elongation l, then tensile force will be

$$F_1 = \frac{YAx}{L} \quad \text{--- (ii)}$$

The amount of work done for an additional small increase dx in the wire will be

$$dW = F_1 dx \cos 0^\circ$$

$$= \frac{YAx\,dx}{L} \quad \text{--- (iii)}$$

Therefore, work done during the whole increase in length of the wire from 0 to l is obtained by integrating eq (iii). Thus

$$W = \int_0^l \frac{YAx\,dx}{L}$$

$$= \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{1}{2}\frac{YA}{L}\left[l^2 - 0\right]$$

$$= \frac{1}{2}\left(\frac{YAl}{L}\right) \times l$$

$$W = \frac{1}{2} \times F \times l$$

This amount of work done is equivalent to elastic potential energy stored in a stretched wire.

$$\text{i.e.,} \quad \boxed{U = \frac{1}{2} \times F \times l}$$

Also, elastic potential energy stored per unit volume of wire is called energy density, and is given by

$$u = \frac{1}{2} \times \frac{F \times l}{AL}$$

$$u = \frac{1}{2} \times \left(\frac{F}{A}\right) \times \frac{l}{L}$$

$$\boxed{u = \frac{1}{2} \times \text{stress} \times \text{strain}}$$

Also, $Y = \frac{Stress}{Strain}$

or, $Stress = Y \times Strain$

So, $u = \frac{1}{2} \times (Y \times Strain) \times Strain$

$\therefore \boxed{u = \frac{1}{2} \times Y \times (Strain)^2}$

But, $Strain = \frac{Stress}{Y}$

$\therefore u = \frac{1}{2} \times Stress \times \frac{Stress}{Y}$

$\therefore \boxed{u = \frac{1}{2} \times (Stress)^2 / Y}$

Elastic after effect:—

The delay in regaining the original state by a body on the removal of deforming force is called elastic after effect.

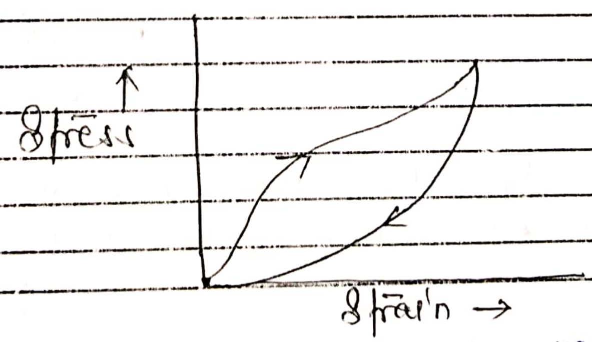
This effect is minimum for quartz and Phosphor Bronze and maximum for glass fibre.

Elastic fatigue:—

It is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

Elastic hysteresis:—

The fact that the stress - strain curve is not retraced on reversing the strain (for a material like rubber) is called elastic hysteresis.



per unit cross-sectional area of a body when the length of the body increases in the